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ABSTRACT

This is a magazine for teachers of mathematics in the South Pacific who teach at the Form I level or above. Most of the articles present activities or materials to use in the classroom. Included are articles on instructional strategies, curriculum developments, interesting problems, puzzles, and math lab activities. This issue includes an article on a UNESCO-sponsored Curriculum Project. It is a joint effort to develop curriculum by and for mathematics educators in the South Pacific territory schools. Background and goals of this Project are detailed. (JP)

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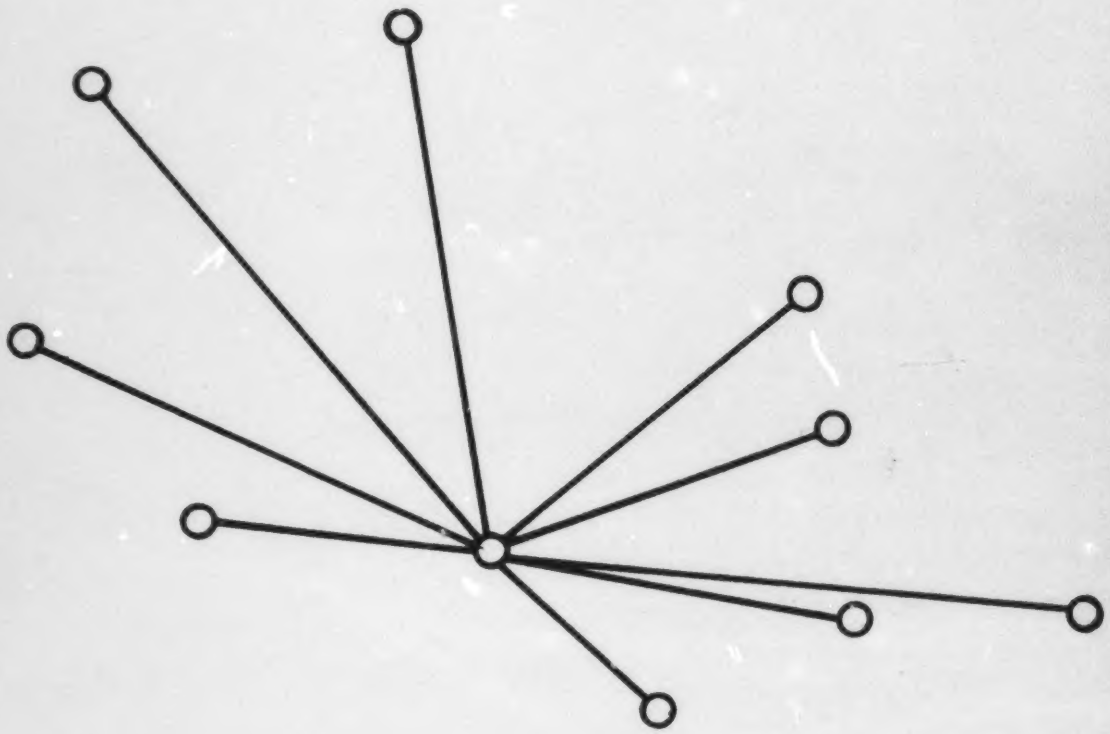
MATHEMATICS



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FORUM

Mathematics Forum is a magazine for all teachers of Mathematics in the South Pacific who teach at the Form I level or above.

It is a cooperative venture:

- The U.N.D.P. Curriculum Development Unit is financing the production.
- U.S.P. is financing the distribution to the various island territories.
- Departments of Education are handling the distribution to teachers within their own territories.

Mathematics Forum does not reflect the views or policies of any Education Department or other Educational body. Any opinions expressed are the opinions of the editors or contributors who are acting as individuals.

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EDITORIAL

It is not so long since the first edition of Mathematics Forum was despatched so there has not been sufficient time to get a reaction from readers concerning its general style and content. The Editorial Committee is very much aware that the magazine did not live up to the objectives outlined in the first editorial. This, the second edition of the magazine, is we believe an improvement in that there is a greater range in the type of article and some discussion and comment more in keeping with the title of the magazine. But there is a long way to go yet.

One way of measuring the success of the magazine will certainly be by the amount of discussion that is included. So may we ask you once again to let us have your views, comments, questions and requests. If you have any suggestions for improving any aspect of mathematics teaching, however simple the ideas may appear to you, there will be many other teachers who will gain from reading about your approach. So please do not hesitate to write to us.

Only if you let us know what sort of magazine you want can future editions of Mathematics Forum meet the needs of the region.

COMPUTERS AT WORK

[Editorial note: This article by Mr. Lionel Yee, who is the Head of the Electronic Data Processing Unit in Suva, is the first of a series of articles designed to explain what an electronic computer does and how it works. At the end of the article are two pages of photographs which indicate the parts of a computer which Mr. Yee refers to in his article. In the article the parts are labelled (A) (B) (C) etc., and the corresponding parts in the photographs are also labelled (A) (B) (C) etc.]

1. What is a Computer?

A computer is an electronic device that has a variety of functions. For instance, it has a section (A) that will accept information (input), it also has another section (B) that will produce information (output). This may appear in many ways, the most common being printed on paper (C). In between, there is a section that can do calculations (D), store information and a large number of other things.

2. The Regional E.D.P. Computer Centre in Suva

At the E.D.P. Computer Centre in Victoria Parade in Suva, a powerful and flexible computer is installed. It consists of a number of cabinets full of electronic equipment. These are placed at various points in a room comparable in size to a large classroom (E). The various cabinets are connected by cables underneath a raised floor. Some of the equipment looks like large tape recorders (F). These are used to record information and behave very much like the music tape recorders.

3. People working with the Computer

A computer is just a tool and without people it will not do anything. So at Suva, there are lots of people who have learnt how to make the computer work. There are the operators

(c) who "drive" the computer; there are the programmers who do the thinking for the computer. In other words, the programmers plan the work in advance so that the computer knows what to do. When the computer does it however, it does so with great speed and accuracy. It never makes a mistake so long as the programmer didn't make a mistake.

4. Type of Work

At Suva, the computer is put to work on a wide range of applications. It pays people's salaries, sends statements for credit accounts, produces trade statistics, does banking deposits and withdrawals and so forth. It can also do complicated mathematical calculations as well. However, in all cases it requires data to work on, and must be told what to do. If wrong data is fed, wrong results will be produced. If it was told to calculate in a wrong fashion, it will do as it is told. Before the computer starts to work, data must be prepared and this usually takes more time. Usually data is entered using punched cards i.e. oblong cards with oblong holes at various places(5).

5. A typical day at the Computer Centre

At the beginning of each day, the operator has to clean the equipment to ensure it is free of dust. He puts paper on the printing cabinet and then switches on the power. At another section of the Centre the punched cards are prepared from documents or forms from various oragisations or departments. These are put together with instructions written by the programmers. Another person will then ensure all the data is available for the operator.

From this the operator will put the correct data on the computer and the correct buttons are pressed. He follows the instructions accurately and after a very short time the results are printed out. These are then registered and stamped ready

for despatch to the people who use the information.

6. Some Advantages of Computers

Many advantages are found in getting the computer to do certain kinds of work. For example work that takes a long time to do, or work where there is a lot of data and it is difficult to produce accurate statistics speedily. Sometimes the type of work is boring, simple and repetitive. The computer introduces new work habits for people. It has created many new types of work jobs. It has the advantage of doing things accurately and quickly. It also does not get annoyed if you give it too much of the same type of work. In fact, it enjoys it. It likes doing complicated things all the time and does not get tired. It does not mind being bossed around and always obeys your instructions. In other words you could call it "The Speedy Idiot".

L.D.S. YEE, SUVA.

KEY TO PHOTOGRAPH

STORAGE (F)

STORAGE (F)

CALCULATING SECTION (D)

(G) OPERATOR

STORAGE (C) (B)

STORAGE (C) (B)

TROLLEY

INPUT (H) (A)

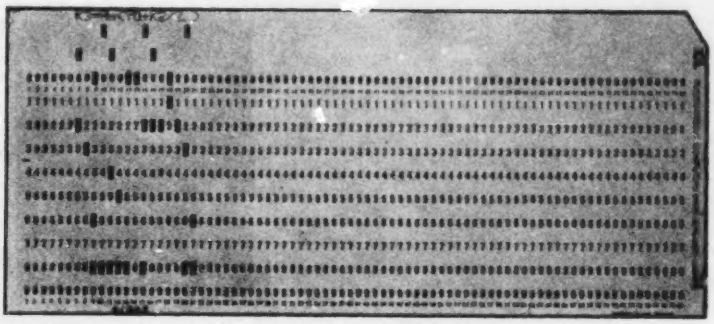
OUTPUT PRINTER (C)(B)

NOT SHOWN IN PHOTOGRAPH

OUTPUT PRINTED ON PAPER
(B) (C)

INFORMATION ENTERED
ON PUNCHED CARDS.
(A) (H)

PUNCHED CARD. (H)



PLAYING WITH 9

A favourite pastime of many children and teachers is developing tricks with numbers. No doubt you have seen and used tricks such as, 'Think of a number between 0 and 10; double it . . . '.

Below are several tricks with the number 9.

1. Take any number, say 936, and multiply it by 9:

$$936 \times 9 = 8424.$$

Add the digits of the answer:

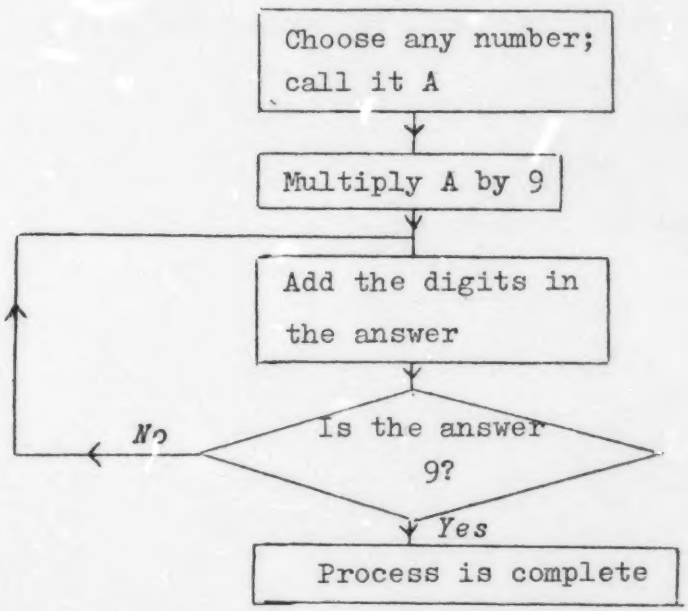
$$8 + 4 + 2 + 4 + 18.$$

Add the digits of this answer:

$$1 + 8 = 9.$$

The result is 9.

The procedure is illustrated by the following flow chart:



Questions:

1. Begin with the following numbers, and work through the flow chart for each:

   83, 142, 836, 1029.

   Is the final answer always 9?

2. Why is the final answer always 9? Can you justify the process algebraically?

Here is another trick with 9:

2. Write down the numbers 12345679 (omitting 8).
   Multiply the number by 9 x 2:

   12345679 x (9 x 2) = 222 222 222.

Questions:

1. Multiply 12345679 by 9 x 3. What is the answer?

2. Under what conditions does the procedure produce
   (a) a series of 4s,
   (b) a series of 5s,
   (c) a series of 6s?

3. Can you explain the procedure?

Finally, here is a third trick. Can you explain the procedure and why it works?

9 x 9 + 7 = 88

9 x 98 + 6 = 888

9 x 987 + 5 = 8 888

9 x 9876 + 4 = 88 888

9 x 98765 + 3 = 888 888

$$9 \times 987654 + 2 = 8\ 888\ 888$$
$$9 \times 9876543 + 1 = 88\ 888\ 888$$
$$9 \times 987654321 + 0 = 888\ 888\ 888$$

There are many other tricks with 9 and other numbers. If you know others, write to the editors of Mathematics Forum.

M.S. Britt, U.S.P.

DIVISION

[Editorial Note: This is an article concerning the lay-out for a standard long division sum.]

Division is repeated subtraction in the same way that multiplication is repeated addition e.g.

16 + 16 + 16 + 16 + 16 = 16 x 5 = 80 16 added together five times = 80	80 ÷ 16 = 5 16 can be subtracted from 80 five times.

In doing a division problem it is important to understand that we are seeing how many times we can subtract the divisor from the dividend. In doing this we remember the place values of our number system.

10,562 ÷ 8

8 can be subtracted 1000 times from 10,562.

```
8 ) 10,562 |
    8,000  | 1000
    2,562  | ------>  This leaves a remainder of 2562.
    2,400  |  300 --> 8 goes into this (can be subtracted from)
      162  |          300 times.
      160  |   20 --> This has a remainder of 162
        2  | 1320 --> 8 goes into 162 20 times, remainder of 2.
```

10,562 ÷ 8 = 1320 + 2 remainder.

In the division above we first saw how many 1000's of 8 could be subtracted from 10,562. Then we saw how many 100's could be subtracted from the remainder. It was 300 times. We then saw how many 10's of 8 could be subtracted from 162. It was two 10's or 20. So we come down the number system 1000, 100, 10, 1. Here is another example:

$56732 \div 26$

56732	
52000	2000
4732	
2600	100
2132	
2080	80
52	
52	2
..	2182

Answer

The big advantages of this method are that it is based on understanding and that, even if the multipliers at any stage are not as big as they might be, this will not effect the final answer. This latter point is illustrated by an example in another base.

Consider 1432 divided by 34 in base 5, or in short

$1432_{(5)} \div 34_{(5)}$

1432		
340	10	→ 34 can be subtracted from 1432 at least 10 times
1042		
340	10	→ 34 can be subtracted from 1042 at least 10 times
202		
123	2	→ 34 can be subtracted from 202 2 times
24	22	

Answer is $22_{(5)}$ with remainder $24_{(5)}$. We can check this carrying out the inverse operation.

$$\begin{array}{r} 34 \\ \times\ 22 \\ \hline 123 \\ 1230 \\ \hline 1403 \\ +\ \ 24 \\ \hline 1432 \end{array}$$

Here is another example.

$1221_{(3)} \div 12_{(3)}$

$$\begin{array}{r|l} 12\ \overline{)\ 1221} & \\ 1220 & 100 \\ \hline 0021 & \\ 12 & 1 \\ \hline 2 & 101 \end{array}$$

Answer $101_{(3)}$ remainder 2.

Again we check by carrying out the inverse operation.

$$\begin{array}{r} 101 \\ \times\ \ 12 \\ \hline 202 \\ 1010 \\ +\ \ \ 2 \\ \hline 1221 \end{array}$$

This is not really a "new" method of division but an "expansion" of the traditional method as you will see by comparing the two methods. Here is the traditional way of setting out the sum 56732 ÷ 26

```
      2182
   ------
26 ) 56732
     52
     ---
      47
      26
      ---
      213
      208
      ---
       52
       52
       --
       ..
```

In this method we said
How many times does 26 go into 56?
The answer is 2.
What we should really be saying is, "How many times is 26 going into 56732?"
The answer is 2000 times,
and so on.

I believe that if we use the method outlined in this article our children will get a better idea of what division is all about.

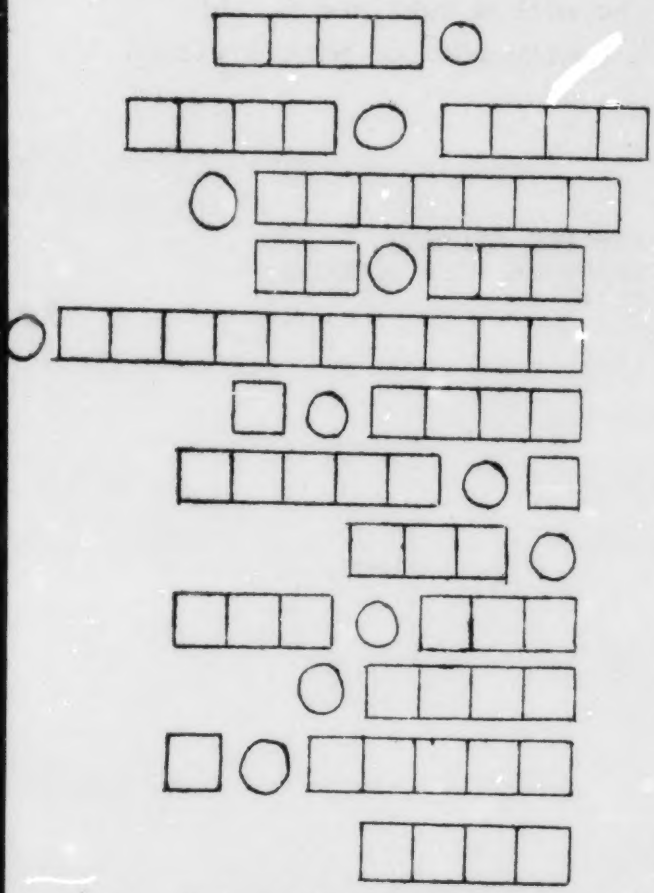
BROTHER GILBERT,
VATUWAQA BOYS SCHOOL,
FIJI.

SCRAMBLED WORDS

[Editorial note: This article is reproduced by kind permission of the Editor of the Nauru Maths Teacher. If you or the children you teach would like to make up similar puzzles which relate to any other mathematical topics we would like to see them.]

Unscramble each word. Each one is the name of a shape or a solid.

Write the answers in the space at the side.



S P I R M	
L A R G E C E N T	
L E G T R A I N	
R S H E E P	
S E M I - R E L I C C	
C L E R I C	
S E E P I L L	
C E B U	
P R I M D A Y	
A R E S Q U	
D R Y C L I N E	
O N C E	

Write the circled letters in the spaces below and you will find a message.

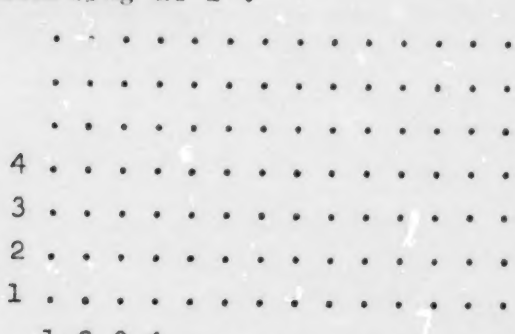
A LESSON ON PRIME NUMBERS

Lesson outline

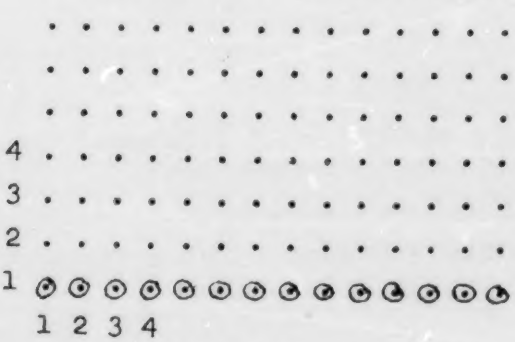
We outline a lesson in which the definitions can be derived after class activity. Each student is given a square of dots (lattice) say 20 x 20. This can be done by marking the intersection of lines on square paper.

Teacher's instructions or questions will be labelled "T", the students' expected response by "S".

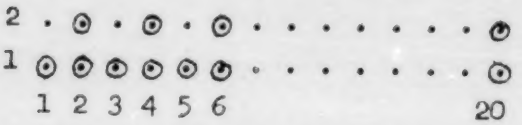
T: "Label your rows and columns horizontally and vertically starting at 1".



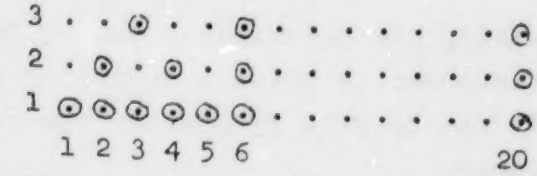
T: "Circle each dot in the bottom row"



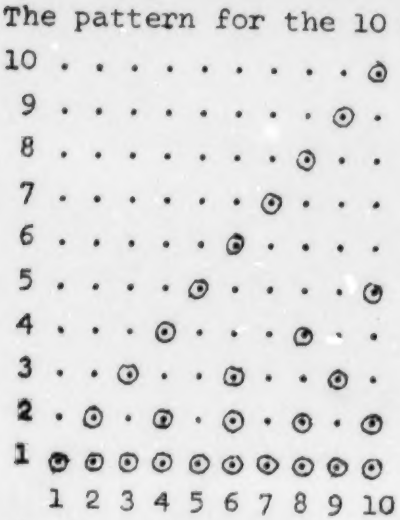
T: "In the second row, leave the first dot, circle the next one, leave one, circle one, and so on till end of the row".



T: "In the third row, leave <u>two</u> dots, circle one, leave two, circle one and so on".

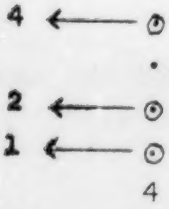


T: "Row 4, leave 3 dots circle 1 dot and so on.
Row 5, leave 4 dots circle 1 dot and so on.
Continue till every row has had some dots encircled (after some stage there will be only one circle in a row)".
The pattern for the 10 x 10 square will look like this:



Note: This can look very effective if different colours can be used for circles in different rows.

T: "Let's take a number on the horizontal line. 4 for instance. There are 3 encircled points above it. What numbers do these points have to the left?"



S: "1, 2 and 4".

T: "How are 1, 2 and 4 related to 4?"

S: "They are factors".

T: "Now we take 5 on the horizontal line. How many encircled points?"

S: "Two".

T: "What numbers do we find to the left of these two points?"

S: "1 and 5".

T: "How are 1 and 5 related to 5?".

S: "They are factors".

T: "What are the factors of 9?".

S: "1, 3 and 9".

T: "How can you see this?".

S: "By looking at the encircled points in the 9 column".

T: "We say that the set 1, 3 and 9 is the factor set of 9 and we will write $F_9 = \{1, 3, 9\}$ What is F_{12}?"

S: "-------

T: "Now in your exercise books rule three columns and complete what I am starting, using the diagram. Leave the third column blank".

Number	Factor Set	
1	$\{1\}$	
2	$\{1, 2\}$	
3	$\{1, 3\}$	
4	$\{1, 2, 4\}$	
5		
-		
-		
-		
-		
20		

T: "Good, now you have all finished I want you to label the third column 'Number of members in factor set'. I'll do the first 4 numbers for you".

Number	Factor Set	Number of members in Factor Set
1	{1}	1
2	{1, 2}	2
3	{1, 3}	2
4	{1, 2, 4}	3

Check that when the table is completed that members in the first and third columns are:

Number	No. of Factors	Number	No. of Factors
1	1	11	2
2	2	12	6
3	2	13	2
4	3	14	4
5	2	15	4
6	4	16	5
7	2	17	2
8	4	18	6
9	3	19	2
10	4	20	6

T: "What is the most common number of factors?"

S: "2".

T: "Which numbers have 2 factors"

S: "2, 3, 5, 7, 11, 13, 17, 19".

T: "These are called prime numbers. Name me some prime numbers bigger than 20".

S: "23, —"

T: "Good".

T: "Is 1 a prime number?".

S: "No".

T. "Why not?".

S: "It does not have two different factors".

T: "A number, like 4 or 6, which has more than 2 factors is called a composite number. Name me some more composite numbers".

S: "9, 10, ---"

T: "Good".

T: "Is 1 a composite number?".

S: "No".

T: "Why not?".

S: "----".

Conclusion

T: "We have seen that there are three types of numbers. The prime numbers 2,3,5,7,—; they have 2 different factors. The composite numbers which have more than 2 factors. And 1 which is neither prime nor composite".

[You may wish to compare what has been outlined above to the method in "Pacific Maths" Book 6, Unit 29, Sets 2,3,4, (Page 177). Note that using our lattice the factor sets required in Set 3 can be easily written down.]

Further suggestions:

(1) The numbers 1,4,9 have an odd number of factors. Write down some more. What property do all these numbers have?

(2) $F_{18} = \{1,2,3,6,9,18\}$

$F_{24} = \{1,2,3,4,6,8,12,24\}$

$F_{18} \cap F_{24} = \{1,2,3,6\}$

How are 1,2,3 and 6 related to 18 and 24?

6 is the largest member of the set.

How is 6 related to 18 and 24?

(An introduction to h.c.f.)

(3) Look along the rows rather columns. In row 2 we have the numbers 2,4,6,8,10,12,14,16,18,20. But it does not stop here since by making the lattice big enough we can include every even number or multiple of 2.

Write $M_2 = \{2,4,6,8,10,12 - - -\}$

$M_4 = \{4,8,12,16,20,24 - - -\}$

Similarly $M_6 = \{6,12,18,24,30 - - -\}$

$M_4 \cap M_6 = \{12, 24, 36 \text{ ---}\}$

Is $M_4 \cap M_6$ a set of multiples? Being an infinite set it has no highest member. However 12 is its least member.
How is 12 related to 4 and 6?

IS 1 A PRIME NUMBER?

Appendix:

It is clear from the lesson outlined above that we have defined numbers in such a way that the answer is clearly "No". However many of us were brought up to believe that 1, 2, 3, 5, 7 and so on are the prime numbers and many dictionaries give 1 as an example of a prime number. The definition given might be as follows:

A (natural) number is prime if its only factors are 1 and the number itself. A number is composite if it is not prime.

According to this definition 1 has 1 and itself as its only factors and so is prime.

What possible objection can there be to 1 being a prime number? The objection can be regarded perhaps as only a theoretical one. The Greeks over 2000 years ago in considering the nature of composite numbers saw that they could be expressed as the product of prime numbers. For example 120 is composite and we can write

$120 = 2 \times 2 \times 2 \times 3 \times 5.$

Provided we write this product with the prime numbers in increasing order, this is the only way we can express it. The statement that every composite number can be expressed in only one way as a product of prime numbers in increasing order is known as the Fundamental Theorem of Arithmetic.

This may appear obvious, but take a large number, say 8633. Is it prime? You may like to try and find prime factors before reading on. One of the following is true

$8633 = 83 \times 101$ —— (1)

$8633 = 89 \times 97$ —— (2)

Check which one by multiplying. The fundamental theorem says that we cannot have (1) and (2) both true. Now if 1 is prime then there is a problem. We can now express a composite number in many different ways as a product of primes.

$6 = 2 \times 3$

$6 = 1 \times 2 \times 3$

$6 = 1 \times 1 \times 2 \times 3$

$6 = 1 \times 1 \times 1 \times 2 \times 3$, and so on.

This violates the fundamental theorem of arithmetic; it is the historical reason why we do not wish 1 to be prime.

J. PORRIT, U.S.P.

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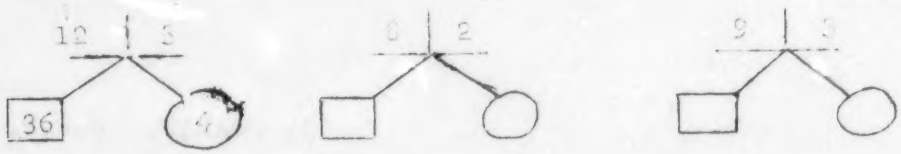
PUZZLES FOR TEACHING

[Editorial note: In the May issue of Mathematics Forum we included some extracts from a letter written by Mr. D. McLeod of Nauru. Below are some further extracts from his letter. We are sure that the ideas contained in his letter will be of interest to teachers looking for ways to enliven their lessons.]

A Find the rule in the first puzzle. Use it to complete the others:



B Many years ago, people learnt their tables only as far as the five times table. Here is how they multiplied the other numbers less than 10:

Multiply 9 x 7

Write the numbers like this

$$\begin{array}{l} 9 \\ 7 \end{array}$$

Subtract each number from 10, and write the answer alongside. Draw in the cross X, and multiply 1 by 3

$$\begin{array}{l} 9 \quad 1 \\ 7 \quad 3 \\ \hline 9 \times 1 \\ 7 \quad 3 \\ \hline \quad 3 \end{array}$$

Subtract along either line (9 - 3 or 7 - 1).

$$\begin{array}{l} 9 \times 1 \\ 7 \quad 3 \\ \hline 6 \quad 3 \end{array}$$ answer

Use the method above to do these:

(a) 9 x 8 (b) 8 x 8

(c) 7 x 8 (d) 7 x 7

Can you extend this method to multiply numbers larger than 10 but less than 20, e.g. 13 x 15.

C Here is a code puzzle.

A	D	E	L	N	P	R	S	T	U
120	72	210	60	280	315	378	180	385	330

Work out these problems. Use the code to find the letters and write them into the square.

1. 70 x 4
2. 9 x 20
3. 18 x 10
4. 36 x 5
5. 35 x 6
6. 42 x 5
7. $\frac{360}{2}$
8. $\frac{630}{3}$
9. $\frac{770}{2}$

1	2	3
4	5	6
7	8	9

Make up a puzzle like the one above.

D. Mc LEOD, NAURU.

COORDINATES

[Editorial note:

Mr. McLeod of Nauru secondary school sent to the U.N.D.P. Mathematics Section comments and notes on the pupil's pamphlet COORDINATES I some of which are reproduced below. Even if the pamphlet has not been seen the suggestions will be understood.]

According to Dienes, to teach a concept we should do three things:-

A. Present it in as many disguises as possible i.e. change the visual form.

This has been done through the use of

(i) geo-boards,
(ii) the classroom,
(iii) graph paper,
(iv) and maps.

B. Change all the mathematical variables possible. In this pamphlet the concept being taught is the use of of coordinates to fix a point. Possible mathematical variables that could be changed are:-

(i) locating an area rather than a point,
(ii) changing the scale,
(iii) changing the surface,
(iv) changing the number of dimensions,
(v) changing to polar co-ordinates.

C. Present an inverse case if it exists.

B (i) Locating an area rather than a point i.e. labelling the columns instead of the lines (cf. bar graphs)

EXAMPLE I

Secret Codes

5	A	B	C	D	E
4	F	G	H	I	J
3	K	L	M	N	O
2	P	Q	R	S	T
1	U	V	W	X	Y
	1	2	3	4	5

(Z is missing)

1. Can you read these messages?

a. 33 15 52 34 42 44 42 55 15
42 51

b. _ _ _ _ _

c. _ _ _ _ _

2. Write a message of your own.

EXAMPLE II

The book S.M.P. A contains the game of "Battleships" in which regions, as opposed to points, are labelled by coordinates.

B (iv) CHANGING THE NUMBER OF DIMENSIONS

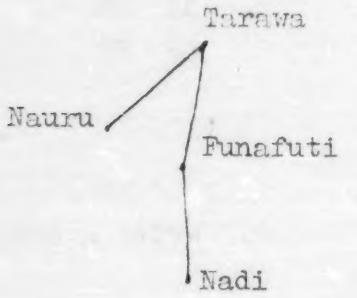
(a) One dimension.

"On a line we need one co-ordinate - the distance from the origin."

"Does the line have to be straight?"

"Will any line (route) do provided it is fixed?"

e.g.



Provided the plane follows the route, we need to know only the distance it has gone to fix its position.

(b) Three dimensions.

e.g. "Where am I in the room? (in paces)."

"Where is my head?"

"Where am I if I am at (87°E, 28°N, 29141 ft)?"

"Where am I now (180°E, 0°S, -2000 ft)?"

TEACHERS' GUIDE (some suggestions)

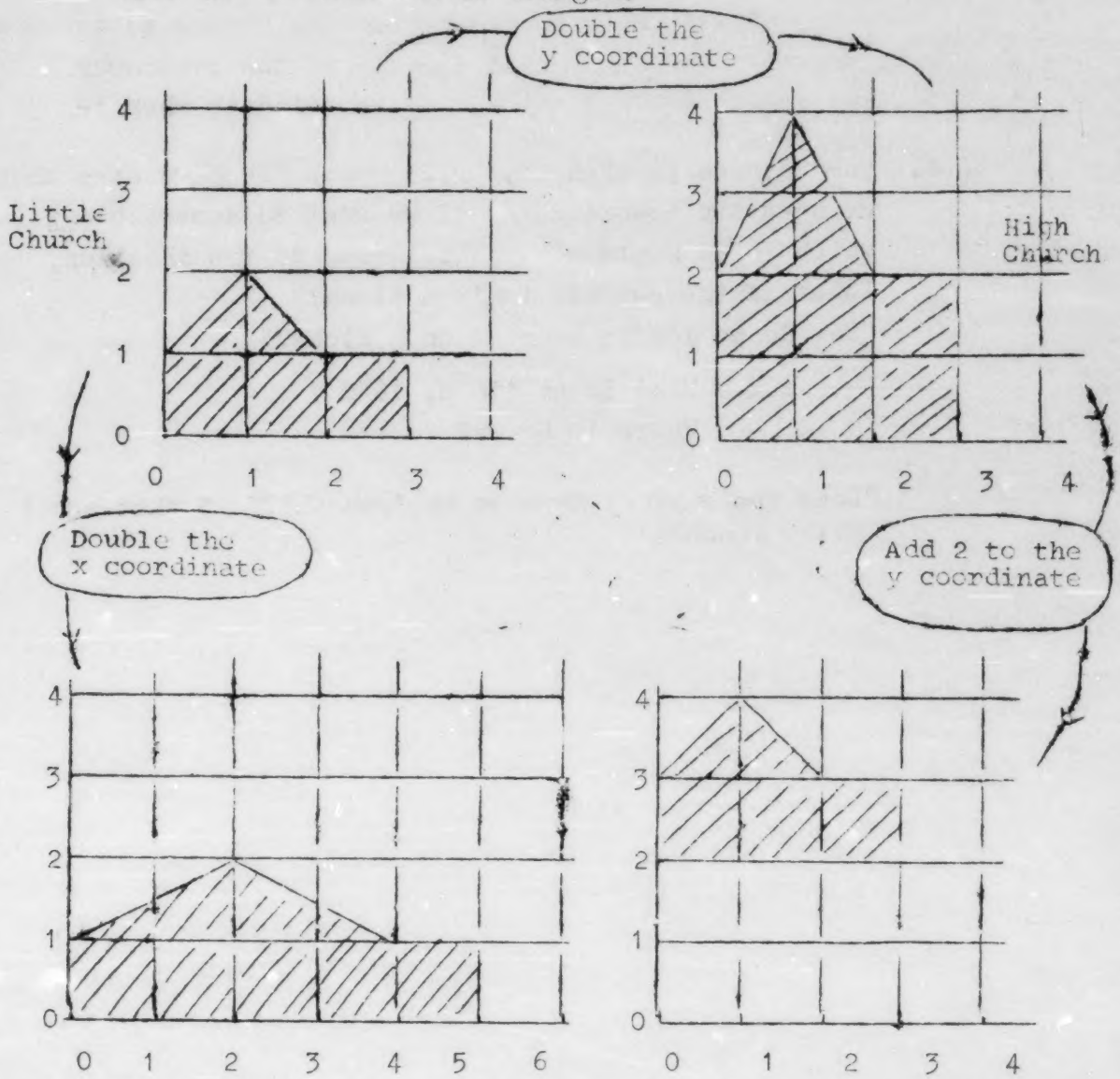
One of the things we should always look for in our teaching is an opportunity to tie our work in with other subjects. P. 23 of the unit offers the chance to discuss with the students (or enlist the aid of the geography teacher in discussing) the use of coordinates when :-

i. The surface is changed. P.23 opens "We have seen that we need two co-ordinates if we want to describe a point which is in a plane" . This leads to the question, "What if the surface isn't a plane?"
"How do we locate a point on a globe?"

e.g. What is at $178^{\circ}E$, $18^{\circ}S$?
Where is Nauru?

"Does the surface have to be 'smooth'?" " What about hilly ground?"

References. Pearcy and Lewis. "Experiments in Mathematics, Stage 1" contains a glorious illustration of what happens when one co-ordinate is changed.



Low church

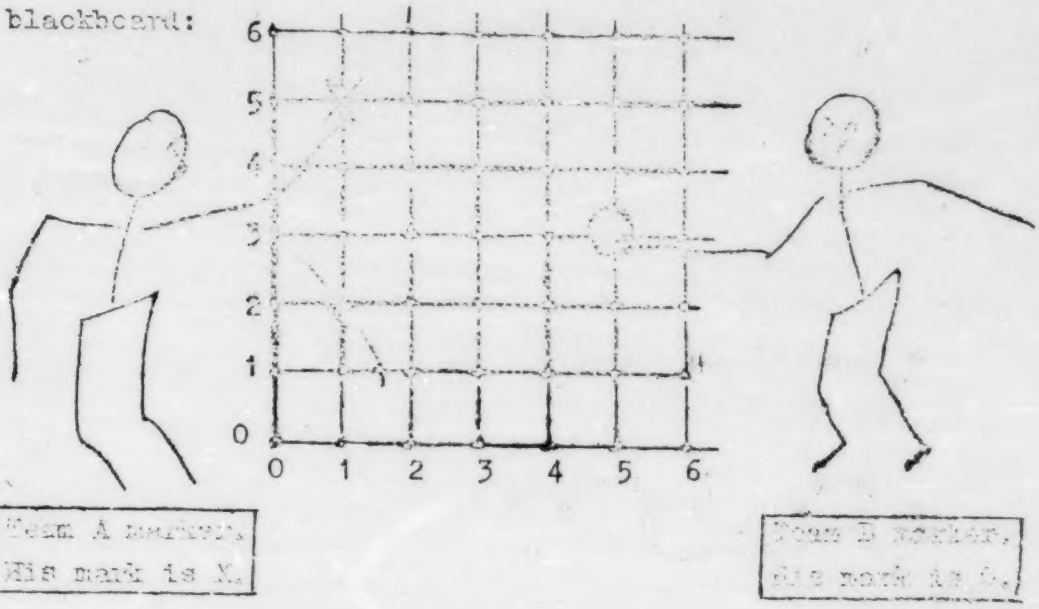
Little church on its way to heaven.

The same book contains a suggestion for a "snakes and ladders" type game using co-ordinates.

[Editorial note: Mr. McLeod also comments on the game Tic-Tac-Toe which occurs on page 13 of the pamphlet Coordinates I. To help readers here is a reproduction of the page.

TIC-TAC-TOE

You can play this game with two teams. A and B. First you need a diagram like this on the blackboard:



Each team has to try to get 4 'X's or 4 'O's in a line. The lines can be along a row, down a column or along a diagonal, but the 'X's or 'O's must be next to each other.

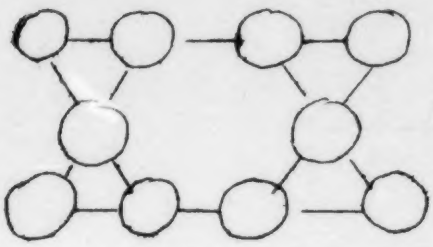
Each team has turns to choose points. The pupils must use coordinates to name points. There must be no shouting, so when the teacher asks your team to name the next point, put your hand up if you know a good point.

Each team can choose its own marker.

The game of Tic-Tac-Toe is important not because of what it teaches about co-ordinates, but because it is asking for a logical plan of attack.

- Decide on a plan.
- Try it. Does it work?
- If not, decide on another plan.
- Try it etc.

When the children are sure they have the answer and can prove it in practice - against the teacher on the board for instance - you might try them on this game which is taken from a recent copy of " Mathematics Teaching" (U.K.)



(3 O's or X's in a row wins)

Where is the best place for the first player to go? Why?
Where MUST the second player reply?
Where does the first player go next?
Where MUST the second player?

NUMBER PATTERNS

[Editorial note: Mr. McLood of Nauru Secondary School has sent to the U.N.D.P. Mathematics Section some suggestions for additions to the U.N.D.P. pamphlet "Number Patterns I". Among the suggestions are those below. Even if readers have not seen the pamphlet the ideas will be understood.]

ADDITION THE EASY WAY

This is an ideal trick for a new form 1. ("Adding up the High School way").

Tell the class you are tired of adding the old way, and that from now on you will put the answer first.

(a) Have a student call out a 4, 5 or 6 figure number e.g. 3 5 8 7 6 5

(b) Leave four lines and write down the answer. (Subtract 2 from the units column and put a 2 in front)

(c) Have another student call out a number and write it up e.g. 482936

```
  3 5 8 7 6 5
  . . . . . .
  . . . . . .
  . . . . . .
  . . . . . .
-------------
2 3 5 8 7 6 3
-------------

  3 5 8 7 6 5
  4 8 2 9 3 6
  . . . . . .
  . . . . . .
  . . . . . .
-------------
```

(d) Complain that he called it too quickly, slowly, softly or whatever and show the class how it should be called as you write up the next line. Make sure every column in his line and yours adds to 9.

	3	5	8	7	6	5	
	4	8	2	9	3	6	His
	5	1	7	0	6	3	Yours
	.	.	.	.	.	.	
	.	.	.	.	.	.	
2	3	5	8	7	6	3	

(e) Repeat these last two steps.

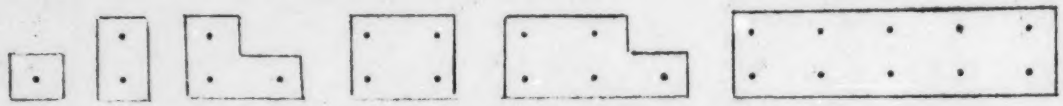
	3	5	8	7	6	5	
	4	8	2	9	3	6	His
	5	1	7	0	6	3	Yours
	3	8	6	4	5	9	His
	6	1	3	5	4	0	Yours
2	3	5	8	7	6	3	

The class will find that the answer which you put up before the problem is correct.

Try this trick out. Can you explain how it works? Do you know any other tricks of this type?

ODD AND EVEN NUMBERS

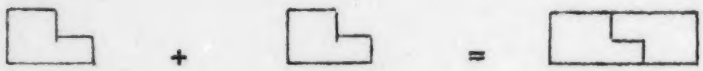
With the less able pupils at least, it might be more rewarding if they made a set of models to test their conclusions e.g.



Then,

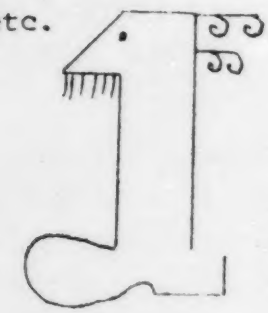
EVEN + EVEN = EVEN

ODD + ODD = EVEN



ODD + ONE = EVEN

etc.

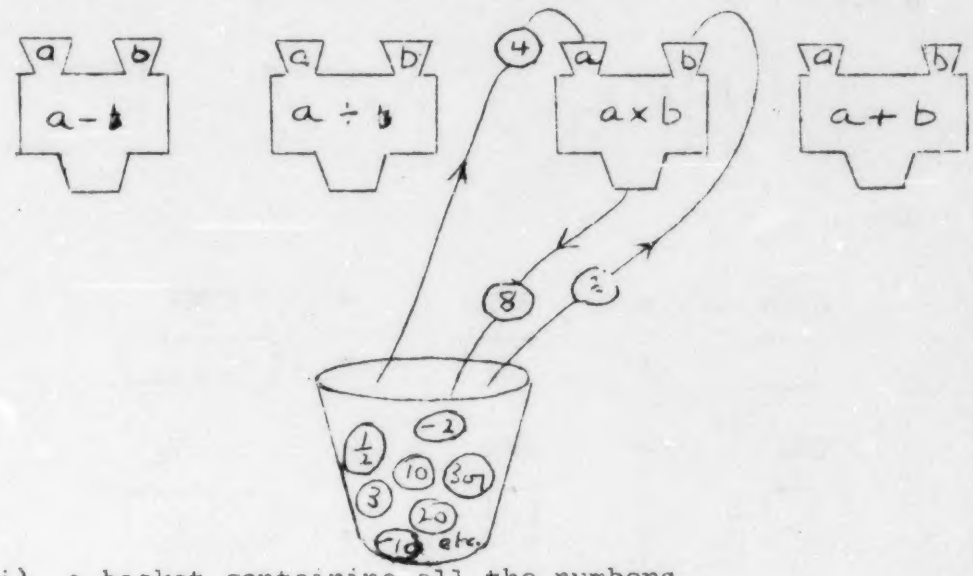


An "ODD" number

D.McLEOD, NAURU.

A NUMBER SYSTEM IN MINIATURE

In ordinary arithmetic there are 4 basic operations - addition, subtraction, multiplication, and division -- and an infinity of numbers that we can use. Perhaps we may picture the whole system like this:



We have

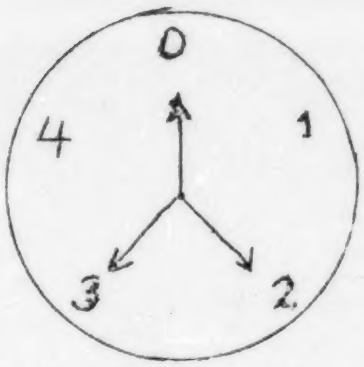
(i) a basket containing all the numbers.

(ii) 4 number machines which do the operations.

We can copy any pair of numbers from the basket, put them through a machine, and the answer is always to be found in the basket. (In fact, there is an exception to this statement; we can't divide by 0, but that's the only exception.) We may think of this basket of numbers and the 4 number machines as forming a complete number system.

Of course it's a big system and we spend a lot of time at school ensuring that children can play the roll of the number machines. However there are other number systems just like this one that we have described but with far fewer numbers. And as such they are much easier to handle.

We will make a number system with just five numbers - 0, 1, 2, 3, 4.



Imagine the numbers equally spaced around the edge of a circle.

To add 2 and 1 we

i) Move a pointer (a pencil will do) from 0 (the starting point) round 2 places clockwise,

ii) move the pointer a further 1 place, and then

iii) check which number the pointer is at.

We see then that $2 + 1 = 3$

Similarly $2 + 3 = 0$

$4 + 4 = 3$

Readers may say that these are unreasonable statements to make because for instance $4 + 4 = 8$, and not 3. So we emphasize that the + sign, when it is used in this small number system, be interpreted in our new way. We will also call it "addition".

As far as multiplication is concerned, an example like 3×2 is thought of as 3 successive moves of 2 places starting from 0. We find then that

$$3 \times 2 = 1$$

This again seems unreasonable, so again we have to interpret multiplication ($\times$) in this new way.

Some things to do:

(1) Complete the addition table

+	0	1	2	3	4
0					
1					
2		3			
3					
4					3

What do you notice?

(2) Complete a mulitplication table

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1			
2	0				
3	0		1		
4	0				

What do you notice?

(3) A shopkeeper, who wants to know the change to give from $5 when a person buys an article for $2.67 (i.e. $5-$2.67), often works out the answer from $2.67 + ? = $5.

In the same way to work out 3-4 in our new number system we can get the answer from

$$4 + \boxed{?} = 3$$

Using the dial and pointer or the table we see that

$$4 + \boxed{4} = 3$$

so $3 - 4 = 4$

Check that you can do any "subtraction". Make a "Subtraction" table.

(4) In the same way we can deal with division. To find $3 \div 4$ we solve $4 \times \boxed{?} = 3$

We know that

$$4 \times \boxed{2} = 3,$$

so

$$3 \div 4 = 2$$

Check that you can do "division".
Are there any exceptions ?

SUMMARY:

We find that we can do any amount of "addition" "multiplication", "subtraction" and "division" in our new number system. Once again it can be pictured as ordinary arithmetic. The difference is that there are only a few numbers and that the operations + x - ÷ have to be interpreted in a different way.

Systems like this are called "Clock Arithmetics" or "Modulo number systems". The one we have looked at is the "Modulo 5" number system.

Some more things to do (in the Mod - 5 system)

(1) Try any calculations:- Fractions, cubes, squares, etc.
(2) Work out $1^4, 2^4, 3^4, 4^4$ What do you notice?
(3) Try inventing and solving some equations in the Mod-5 number system.
(4) Are there things which you can do with the ordinary system which you can't do with the Mod-5 system?
(5) What can you find out about the Mod-5 system?

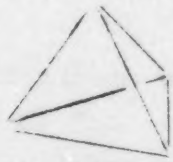
We would like to hear of any observations or discoveries from children or teachers.

E.H. LEATON, U.N.D.P.

PUZZLES

In the May issue of Mathematics Forum four puzzles were given in the article 'Why do so many people dislike mathematics?' Here are possible solutions to those puzzles (There may well be other methods of solution).

1. Make four triangles using six match sticks.



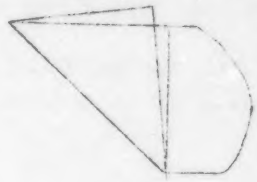
Arrange the match sticks to form a tetrahedron. This has four triangular faces.

2. Make a square by folding an irregularly shaped piece of paper.

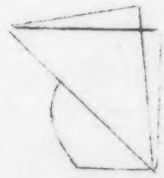
Fold one side of the piece over

Fold up the bottom, and fold down the top to form two right-angles

Take the bottom left hand corner and fold it up so that the left hand edge now lies along the top edge

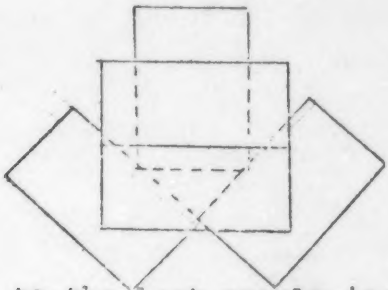


Fold the right hand edge under

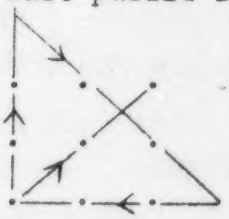


Unfold the uppermost piece to form the square

3. Arrange five match boxes so that any one box touches all of the others. A solution is:

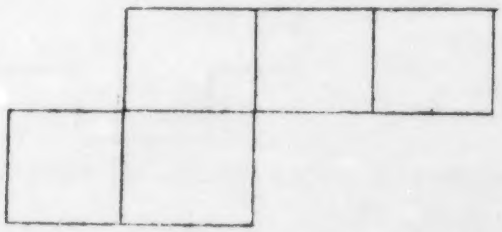


4. A solution to the last puzzle is:



Here are four more puzzles for your amusement.

1. Arrange match sticks to form five squares like this:



Now make the five squares into four squares by changing the position of only two of the match sticks. (There are at least two solutions).

2. This puzzle is an extension of the fourth puzzle in the May issue. How many continuous line segments are needed to traverse sixteen dots arranged in a square?

3. In this multiplication sum

$$\begin{array}{r} SEAM \\ \times \quad T \\ \hline MEATS \end{array}$$

The five letters represent different digits. There is no zero. What are the actual figures?

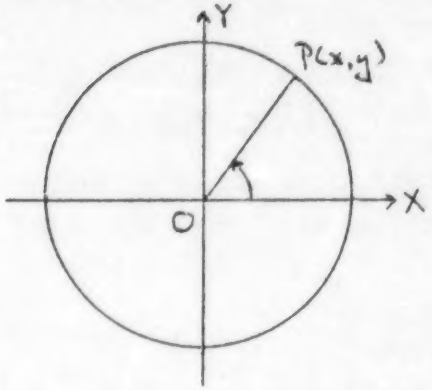
4. Can you cut this cross into five pieces so that one piece is a smaller cross, and the remaining four pieces will fit together to form a square?

5. Arrange six match boxes so that each box touches the remaining five.

ANOTHER LOOK AT TRIGONOMETRY

Traditionally, the trigonometric ratios of sine, cosine and tangent are defined as the ratio of sides of a right-angled triangle. In the following article, the sine, cosine, and tangent of an angle are defined in terms of the coordinates of a point, and the usual trigonometric identities and triangle rules are developed from that point of view.

1. The definition of the sine, cosine and tangent of an angle



P is the point (x,y) on a circle centre the origin, radius 1.

OP = 1

DEFINITION

$\sin \hat{XOP} = \sin X\hat{O}P = y$

$\cos \hat{XOP} = \cos X\hat{O}P = x$

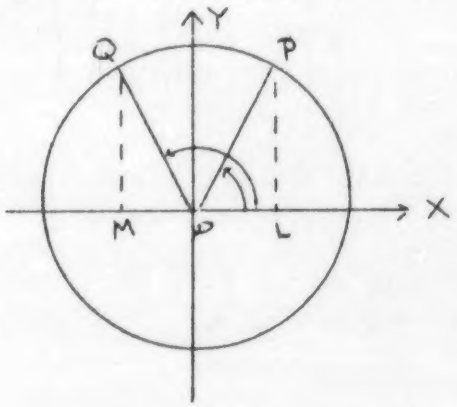
$\tan \hat{XOP} = \tan X\hat{O}P = y/x$

The sine, cosine and tangent of any angle between 0° and 360° may be found practically by using these definitions.

EXAMPLE

Find $\sin 55^\circ$, $\cos 125^\circ$.

Draw a circle of radius 10cm, centre the origin, and mark points P and Q so that $X\hat{O}P = 55^\circ$, $\hat{XOQ} = 125^\circ$.



OP = OQ = 10 cm, and for the purpose of this problem, we require OP = OQ = 1.

Hence the scale for the diagram is 10cm : 1

By measuring OL, LP, OM, MQ, the coordinates of P and Q may be found to be (0.58, 0.82) and (-0.58, 0.82) respectively.

By the definitions then,

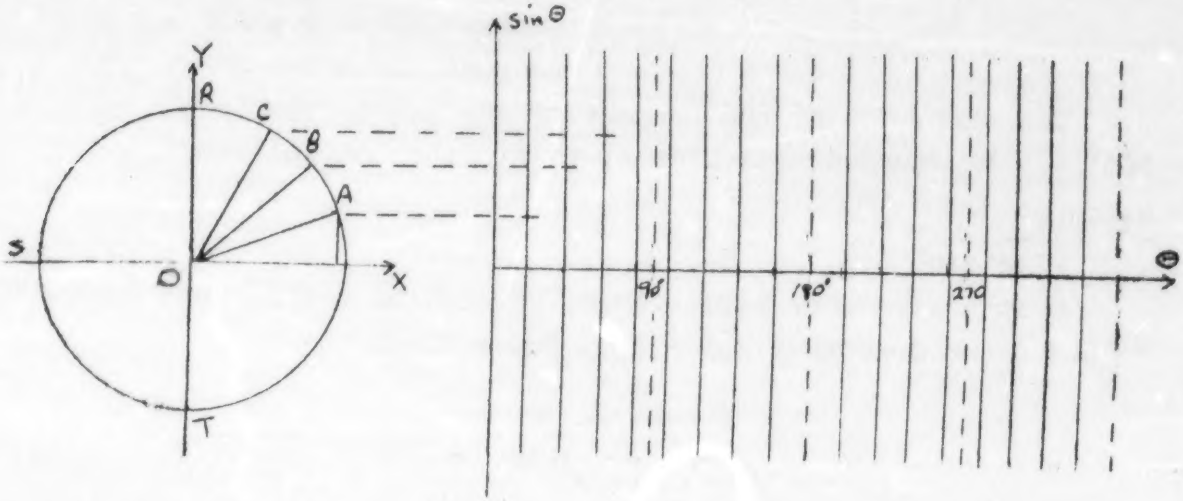
$$\sin 55^\circ = 0.82$$

$$\cos 125^\circ = -0.58.$$

1. <u>The graphs of the sine, cosine and tangent functions</u>

(a) The graph of $y = \sin\theta$, for $0^\circ \leq \theta \leq 360^\circ$

A unit circle is drawn with centre the origin, and points A,B,C.....marked on its circumference so that $X\hat{O}A = 20^\circ$, $X\hat{O}B = 40^\circ$, $X\hat{O}C = 60^\circ$ etc.



A second set of axes, θ and $\sin\theta$ are drawn as shown. Graduate the θ - scale in multiples of 20°.

By the definition of sine,

$\sin 20^\circ$ = y-coord of A

= AL

The distance AL, may be transferred and marked off at 20° on the θ, $\sin\theta$ plane.

Similarly, $\sin 40^\circ$ = y-coord of B

= BM

and this can be marked off at 40°.

This process is continued, and the points marked off joined by a smooth curve to give the graph of $y = \sin\theta$

The coordinates of R are $(0, 1)$, and $\hat{XOR} = 90^\circ$

hence $\cos 90^\circ = 0$

$\sin 90^\circ = 1$

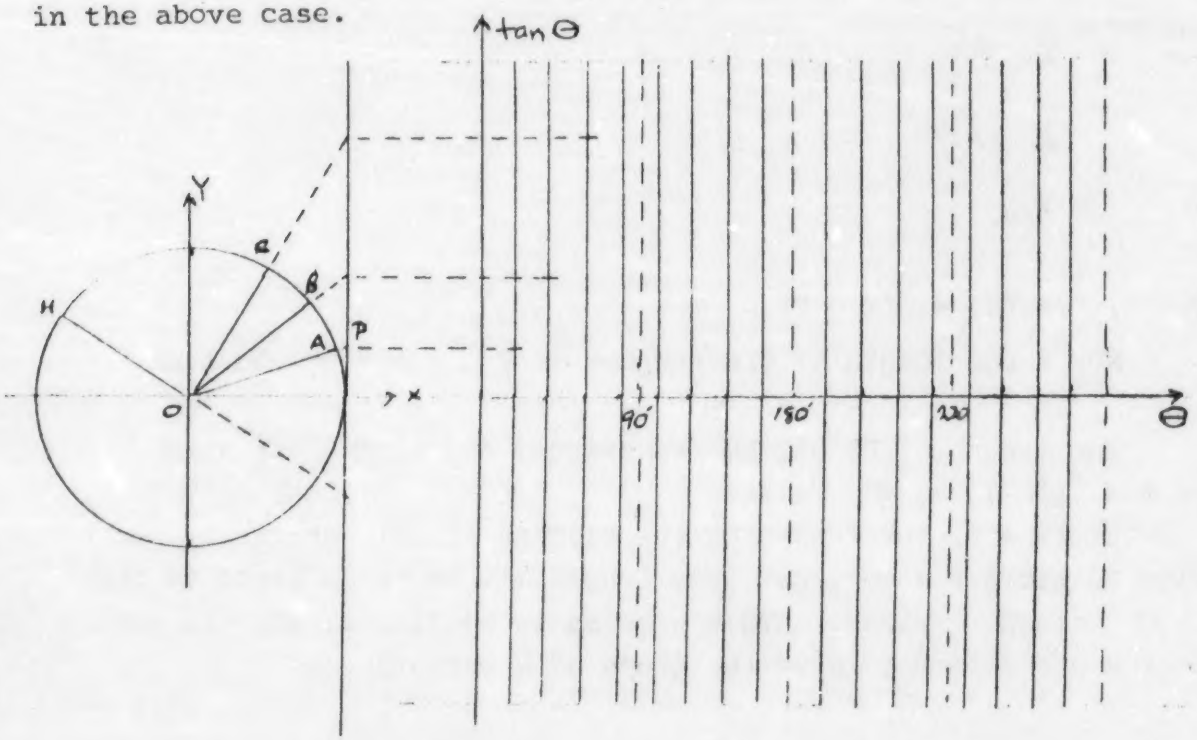
Similarly, the coordinates of S are $(-1, 0)$, and $\hat{XOS} = 180^\circ$

hence $\cos 180^\circ = -1$,

$\sin 180^\circ = 0$

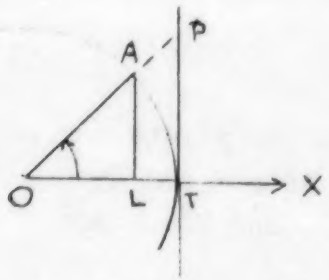
(b) The graph of $y = \tan\theta$, for $0^\circ \leq \theta \leq 360^\circ$.

A unit circle is drawn, and a second set of axes drawn, as in the above case.



If $\widehat{XOP} = 20^\circ$, and A is the point (x,y), then

$\tan 20^\circ = {}^{y}/_{x}$ from the definition of tangent.



If each side of the triangle OAL is enlarged by a scale factor $\frac{1}{x}$.

$\overline{OL} \longrightarrow \overline{OT}$, and $\overline{AL} \longrightarrow \overline{TP}$,

as $OT = \frac{1}{x} \cdot OL$

$= \frac{1}{x} \cdot x$

$= 1,$

and $PT = \frac{1}{x} \cdot AL$

$= \frac{1}{x} \cdot y$

$= {}^{y}/_{x}.$

Hence, $\tan 20^\circ = {}^{y}/_{x} = PT$

PT is the length of the tangent at T cut off by the line $\overleftrightarrow{OA}$.

The length of PT may be transferred and marked off at 20° on the θ , $\tan\theta$ plane.

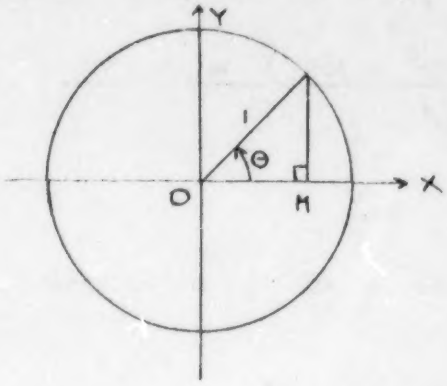
Similarly, the length of the tangent at T cut off by the line OB gives $\tan 40^\circ$, and this length may be transferred to the θ, $\tan\theta$ plane. This process is continued, and the points marked off joined to give the graph of $y = \tan\theta$

(Note: The tangent at T is parallel to the Y-axis, and thus tan 90°, and tan 270° do not exist, as the y-axis, and the tangent at T never intersect).

3. Trigonometric identities

P is the point (x,y), OP = 1, $\hat{XOP} = \theta$

$\sin\theta = y$, $\cos\theta = x$, $\tan\theta = y/x$



The triangle OPM is right-angled, and OM = x, PM = y. Further, by Pythagoras' theorem,

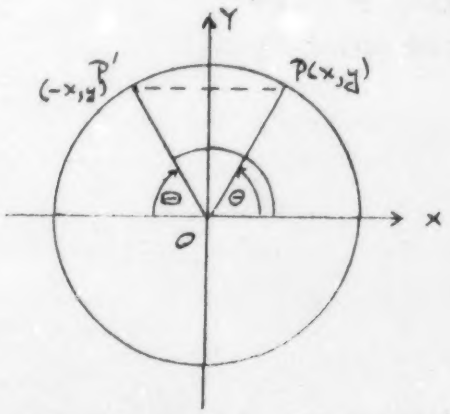
$$OP^2 = OM^2 + MP^2$$

$$\Longrightarrow 1 = x^2 + y^2$$

$$\Longleftrightarrow 1 = \cos^2\theta + \sin^2\theta$$

Suppose M_y is a reflection in the y-axis. The image of P under the reflection is P'.

$$M_y(P) = P'$$



P^1 is the point $(-x,y)$

$\hat{XOP}' = 180^\circ - \Theta$

From the definitions,

$\cos \hat{XOP}' = -x,$

$\iff \cos(180^\circ - \Theta) = -x$

$\iff \cos(180^\circ - \Theta) = -\cos\Theta,$ as $\cos\Theta = x$

$\sin \hat{XOP}' = y$

$\iff \sin(180^\circ - \Theta) = \sin\Theta,$ as $\sin\Theta = y$

$\tan \hat{XOP}' = \frac{y}{-x}$

$= -\frac{y}{x}$

$\iff \tan(180^\circ - \Theta) = -\tan\Theta,$ as $\tan\Theta = \frac{y}{x}$

Similarly, by reflecting P' and P in the x-axis, as identities

$\cos(180^\circ + \Theta) = -\cos\Theta,$

$\sin(180^\circ + \Theta) = -\sin\Theta,$

$\tan(180^\circ + \Theta) = \tan\Theta,$

$\cos(360^\circ - \Theta) = \cos\Theta,$

$\sin(360^\circ - \Theta) = -\sin\Theta$

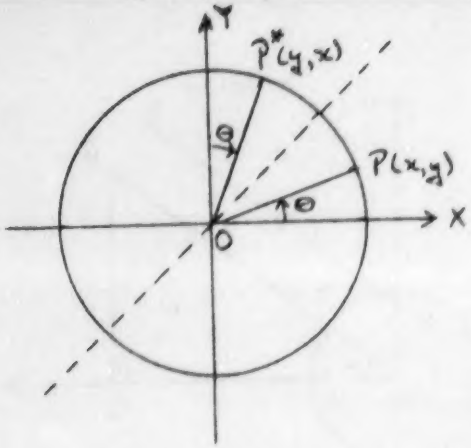
$\tan(360^\circ - \Theta) = -\tan\Theta$

can be obtained.

Now consider reflecting P in the line $y = x$.

Under the reflection $P \longrightarrow P^*$

P^* is the point (y,x).



$\hat{XOP^*} = 90^\circ - \theta$

$\cos \hat{XOP^*} = y$

$\iff$ $\underline{\cos (90^\circ - \theta) = \sin\theta}$ since $\sin\theta = y$

$\sin \hat{XOP^*} = x$

$\iff$ $\sin (90^\circ - \theta) = \cos\theta$ since $\cos\theta = x$

$\tan \hat{XOP^*} = \frac{x}{y}$

$= \frac{1}{y/x}$

$\iff$ $\tan (90^\circ - \theta) = \frac{1}{\tan\theta}$, since $\tan\theta = y/x$

Reflecting P* in the y-axis will lead to the identities

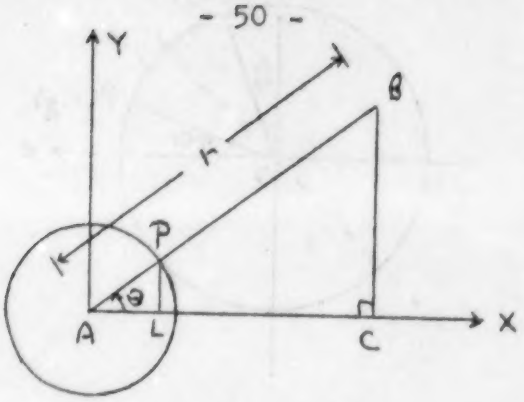
$\cos (90^\circ + \theta) = -\sin\theta$

$\sin (90^\circ + \theta) = \cos\theta$,

$\tan (90^\circ + \theta) = -\tan\theta$,

4. <u>Solution of triangles:</u>

(a) Right-angled triangles:



In the triangle ABC, suppose $\hat{BCA} = 90^\circ$, $\hat{CAB} = \theta$, and AB = r.

At A, draw X and Y axes, so that the X-axis lies along AC.

Draw a unit circle, centre A. The unit circle cuts AB at P.

Let P be the point (x,y),

then $\cos\theta = x$,

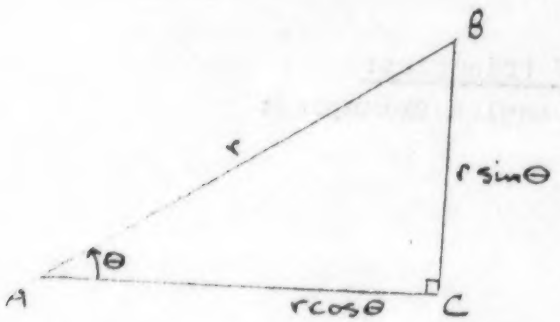
$\sin\theta = y$

The triangle APL enlarges onto the triangle ABC under a scale factor r (since AP : AB = r : 1)

Hence, AC = r.AL = rx = r $\cos\theta$.

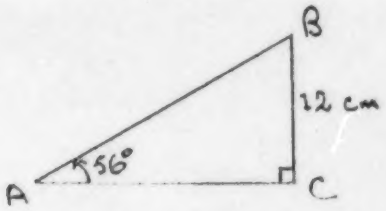
and BC = r.PL = ry = r $\sin\theta$.

These results hold for any right-angled triangle and may be used in solving a right-angled triangle.



EXAMPLE:

In triangle ABC, BC = 12 cm, $\hat{BCA} = 90^\circ$, , $\hat{BAC} = 56^\circ$. Solve the triangle.



(i) $\hat{ABC} = 180^\circ - (56^\circ + 90^\circ)$

$= 34^\circ$

(ii) If AB = r, then BC = r sin 56°

hence r sin 56° = 12 cm

$$\Longrightarrow \quad r = \frac{12}{\sin 56^\circ} \text{ cm}$$

$$= \frac{12}{0.829} \text{ cm}$$

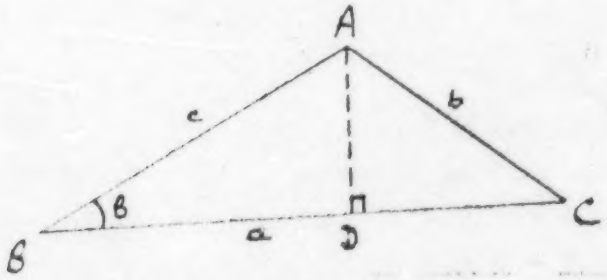
$$= 14.5 \text{ cm.}$$

$AC = r \cos 56^\circ$

$= 14.5 \times .559$

$= 8.1$ cm

(b) Triangles which are not right-angled:



(i) When two sides and the included angle are known.

Suppose AB = c, BC = a, $\hat{ABC}$ = B.

Find b in terms of a, c and B.

Triangle ABD is right-angled,

hence AD = c sin B,

BD = c cos B, and thus

DC = a - c cos B.

Triangle ADC is right-angled,

hence $AC^2 = AD^2 + DC^2$

$b^2 = (c \sin B)^2 + (a - c \cos B)^2$

$= c^2 \sin^2 B + a^2 - 2ac \cos B + c^2 \cos^2 B$

$= c^2 (\sin^2 B + \cos^2 B) + a^2 - 2ac \cos B$

$\underline{b^2 = c^2 + a^2 - 2ac \cos B}$ which is the

<u>cosine rule</u> for a triangle.

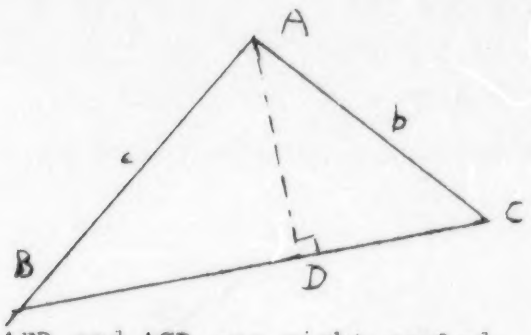
Similarly, $\underline{a^2 = b^2 + c^2 - 2bd \cos A}$,

$\underline{c^2 = a^2 + b^2 - 2ab \cos C}$.

(ii) When two angles and one side are known.

Suppose $A\hat{B}C = B$, $A\hat{C}B = C$, AB = c

Find b in terms of B, C and c.



Triangles ABD and ACD are right-angled. Hence, AD = c sin B, from triangle ABD, and AD = b sin C, from triangle ACD.

Thus, c sin B = b sin C

$$\frac{c \sin B}{\sin B \sin C} = \frac{b \sin C}{\sin B \sin C}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B} \quad \ldots$$

Similarly, by drawing BD perpendicular to AC, it can be shown that

$$\frac{c}{\text{Sin } C} = \frac{a}{\text{Sin } A}$$

Thus, $\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$ which is the sine formula for a triangle.

E.J. BILLINGHURST, U.S.P.

THE U.N.E.S.C.O., U.N.D.P. CURRICULUM DEVELOPMENT UNIT UNIVERSITY OF THE SOUTH PACIFIC

The unit was set up in 1970 and is designed to promote Curriculum Development in all the territories associated with the University of the South Pacific. The unit is financed by the United Nations Development Programme and has twelve team members. Particular fields covered by these advisers are Basic Science, Industrial Arts, English, Commercial Studies, Home Economics, Social Science, Mathematics, Audio-visual methods.

In all fields, new material-which has been written by local teachers and Curriculum Development Unit members - has been produced by the unit and is being tried out in all the territories of the region.

As far as mathematics is concerned the objectives of the unit - which are reproduced from the pamphlet "Introduction to Teachers" are summarized below.

"The basic aim is to provide a service concerning all aspects of curriculum development to the various territories of the South Pacific. We have to take into consideration

- The desirability of setting up a system for continual renewal and appraisal of material and of methods of teaching,
- The somewhat different priorities of development in the various territories,
- the need for local teachers and administrators to be involved in all aspects of development,
- local conditions. Environment, the present stage of teachers' training and the levels of children's understanding of English all have a bearing in the type of

teaching material which we produce,

- the need to co-operate with curriculum developers in all other fields of education,
- the need to produce material which enables teachers to teach well,
- the need to relate the language structure and vocabulary of the material to the English Teaching Programmes.

Do you agree with these aims?"

The Curriculum Development Unit is coordinating the writing of a series of topic units - around one hundred in all. Such a programme enables:

- territories to select whatever materials they feel to be appropriate for their own particular needs,
- schools to work at a pace which is relevant to their children and circumstances,
- territories to select units which lead to any School Certificate Examination, be it Overseas or South Pacific in origin,
- any person to contribute by writing new units, and revising ones which have already been tried. Teachers from different Territories, Mathematical Associations, University and U.N.E.S.C.O. Lecturers are all involved.

Can these methods be improved?

If you would like information about the U.N.D.P. project please write to us here. Your Department of Education will also be able to tell you about the project.

E.H. LEATON U.N.D.P.